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An Interdisciplinary Perspective on Predictive Processing: An Ode to an Innovative STEM Education Scholar

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Abstract

This paper presents an interdisciplinary exploration of the Predictive Processing framework, synthesized through the lens of Professor Uri Leron's pioneering contributions to mathematics and computer science education. We employ a theoretical and applied integration of the Prediction Error Minimization (PEM) principle across diverse domains, reflecting Leron's legacy of bridging intuitive and analytical thinking. The analysis reconceptualizes learning as the dynamic updating of internal "priors," where cognitive resistance is viewed not as a deficit, but as a rational attempt to protect stable mental models. Central to this discussion is the distinction between perceptual inference - updating models to fit reality - and active inference - altering the environment to satisfy internal expectations. These mechanics are further examined through cases of data mining and reinforcement learning in volatile environments, alongside a developmental neurocognitive perspective that frames reading disorders like dyslexia as disruptions in predictive tuning. Shifting from purely mechanistic accounts, the paper further explores aesthetic pleasure and humor, arguing that in these realms, prediction error is not a cost to be minimized but a primary source of delight and insight. By weaving together data science, machine learning, neuroscience, aesthetics, and humor, the paper demonstrates that PEM serves as a robust explanatory framework for human meaning-making. It concludes that expertise is defined not by the absence of errors, but by the sophisticated capacity to transform prediction errors into productive opportunities for deeper understanding, providing a holistic window into the creative and adaptive mind.

1 Introduction

This paper is dedicated to Prof. Uri Leron, an influential and innovative researcher in mathematics and computer science education, known for pioneering initiatives that integrate technology into learning environments (Hazzan et al., 2024). In his earlier work, he is particularly known for his insights into cognitive processes of mathematics learning, the application of computer science cognitive construct, e.g., abstraction and modularity, to

mathematics education, and how it can be supported with technological means (Leron, 1983; Dubinsky et al., 1994; Leron and Dubinsky, 1995). In his later work, Leron was fascinated by modern cognitive theories and investigated their manifestation in learning processes, shedding a new light on how they can be more effectively supported. Specifically, he performed in-depth investigations through the lens of the Dual-Process Theory (summarized in Kehneman, 2002) of cognitive processes during mathematics and computer science education and practice (Leron and Hazzan, 2006; Hadar and Leron, 2008; Leron and Hazzan, 2009). In his recent years, he has dedicated his full attention to the theory of Predictive Processing (e.g., Clark, 2016; 2018), and its potential applications to education and practice (Sorokina et al., 2023).

Uri Leron was born on December 22, 1938, in Kibbutz Nir David. After (almost) 85 years, on December 5, 2023, Uri passed away and was laid to rest in the kibbutz where he was born and to which he remained deeply connected throughout his life. On December 23, 2025, we marked two years since our parting from him. This gathering focused on the cognitive theory of Predictive Processing, which was Leron's most recent research interest.

While Prof. Leron's original specialization was in group theory, in 1980 he decided to shift his focus to research in mathematics and computer science education. His decision to move into educational research was based on his recognition that in this field his contribution would be more significant and distinctive. In retrospect, this transition proved highly impactful, as described above, and left a lasting mark on math and computer science education in several areas, whose far-reaching influence was not necessarily apparent at the time. His research approach and the pedagogical ideas that stemmed from it -- many of which seem self-evident today -- were highly innovative and groundbreaking at the time. In this paper, we examine his most recent research interests, Predictive Processing, present his views and beyond, with several original ideas of our own for its interdisciplinary applications, inspired by his years-long mentorship.

While predictive processing is often associated with neuroscience and computational modeling, its implications reach far beyond these domains. When viewed through a disciplinary lens, for example in education, mathematics, and conceptual learning, the framework resonates strongly with long-standing insights developed by Uri Leron. Leron's work, especially in mathematics and computer science education, offers a rich conceptual bridge between predictive processing and how learners construct, resist, and reorganize knowledge within specific disciplinary contexts. Leron's attention to meaning-making, intuition, and epistemological coherence prevents predictive processing from becoming a purely mechanistic account. In this paper, we examine the expression of predictive processing from an interdisciplinary perspective. This approach is similar to the one implemented by Miller et al. (2022) in the special issue on Predictive Processing and Consciousness.

The paper is structured as follows. First, with respect to Leron's work, we examine the predictive processing framework as a learning theory as well as a central principle underlying the theory of predictive processing, that is, prediction error minimization (PEM). We then examine predictive processing from an interdisciplinary perspective – some highly research-oriented while others are rooted in human emotional experience – considering its roles in data mining, machine learning, neurocognition, aesthetic pleasure, and humor. Finally, we conclude with interpretive thoughts corresponding with Leron's legacy.

2 Predicative Processing Framework through Uri Leron's Lens

2.1 Predictive Processing as a Theory of Learning – Orit Hazzan

In recent years, predictive processing has emerged as one of the most influential frameworks for understanding cognition, learning, and action. At its core, predictive processing proposes that the brain is not a passive receiver of information but an active generator of predictions, continuously anticipating sensory input and updating its internal models based on prediction errors (Clark, 2016; de Bruin and Michael, 2017). Learning, from this perspective, is not primarily about accumulating facts or representations, but about reducing surprise -- refining models of the world so that expectations better align with experience.

Predictive processing reframes learning as a dynamic interplay between prior expectations and incoming evidence. Rather than simply reacting to stimuli, learners bring deeply ingrained priors with them, and when reality deviates from these expectations, a prediction error arises. Learning occurs when the system updates its internal model to reduce future error or comes to the conclusion that the stimulus is misleading and learns to discard it in certain circumstances (Clark, 2018).

Uri Leron's work has long emphasized that learning — particularly in mathematics and computer science — is not about absorbing formal structures, but about making sense of ideas through embodied, intuitive, and epistemological processes. He has argued that learners approach new concepts with powerful intuitive frameworks, often grounded in everyday experience, that shape what feels reasonable, obvious, or even possible (e.g., Leron and Hazzan, 2006; 2009; Hadar and Leron, 2008; Leron and Ejersbo, 2021).

In this sense, the predictive processing framework resonates deeply with Uri Leron's work on building bridges between intuitive and analytical thinking. For Leron, learning was never a simple replacement of "incorrect" ideas with "correct" ones, but a gradual process of mediation between different modes of understanding. Errors, approximations, and initial intuitions were not obstacles to be eliminated, but essential anchors for meaning-making. Similarly, predictive processing emphasizes that when prediction errors threaten stable

mental models, learners tend to protect their existing beliefs rather than revise them. The deliberate construction of bridges, as Uri's proposed – through activities, intuitive representations, and technological tools – allows prediction errors to be transformed from sources of threat into opportunities for learning, enabling intuition to be leveraged rather than suppressed on the path toward deeper analytical understanding.

For example, in mathematics education, learners often resist abstract formalism because it clashes with intuitive expectations about continuity, causality, or magnitude. Leron's analyses show that such resistance is not a deficit but a rational response within the learner's predictive framework. Predictive processing helps us see this resistance as the system protecting a stable model rather than failing to learn.

Learners may encounter prediction errors constantly, but unless those errors are framed as relevant, trustworthy, and productive, they are unlikely to drive change. Leron's work highlights the role of instructional design in creating situations where learners cannot easily dismiss anomalies and are instead invited to reconstruct their models. From a predictive processing standpoint, this is precisely the moment when the system chooses to update its priors rather than suppress the error.

Predictive processing, when viewed through this lens, becomes a disciplinary theory of cognition: learners internalize not only content, but also the predictive structures that define what explanations are acceptable within a field. This perspective helps explain why transferring knowledge across domains is so difficult — predictions that work well in one disciplinary context or social context may fail in another.

Direct communication, for example, is often rewarded in engineering or startup cultures, where efficiency and clarity signal competence, yet the same style can be perceived as insensitive or inappropriate in diplomatic, educational, or culturally diverse settings. Similarly, the belief that hard work reliably leads to recognition tends to hold in school or early career environments but frequently fails in organizational or leadership contexts, where visibility, timing, and social dynamics play a decisive role. In physics education, students often predict that motion requires a continuous force, a model that works well in everyday contexts dominated by friction but fails in formal Newtonian mechanics. What appears as a misconception is, from a predictive perspective, a reasonable model transferred from one context to another where its underlying assumptions no longer hold. These mismatches do not reflect flawed reasoning, but rather context-sensitive predictive models that were once successful and are now being applied beyond their original domain.

2.2 Prediction Error Minimization – Irit Hadar

A central principle underlying the theory of predictive processing is prediction error minimization (PEM) (de Bruin and Michael, 2017; 2018). This principle relies on the notion of predictive processing, conceptualizing the brain as a probabilistic inference system, which attempts to predict the input it receives by constructing mental models of the possible causes of this input (Clark, 2016). Once the input is received, it is compared to the prediction which preceded the input. The brain then aims to minimize the prediction error, namely, the difference between the predicted and the actual input. If the prediction error is small, then there may be no need to revise the model that gives rise to the prediction. If, on the other hand, the prediction error is large, then it is likely that the model fails to capture the causes of the inputs and, therefore, must be revised. In this case, there are two optional courses of action for reducing the prediction error. The first is to revise one's model of the world until the prediction error is satisfactorily diminished (termed 'perceptual inference') and the second is to change the world (e.g., by manipulations that yield additional inputs) so that it matches the model (termed 'active inference').

Let us take as an example the following daily scenario: You come home and expect your keys to be in their usual spot on the hallway table. Your internal "mental model" generates a top-down prediction of what that table should look like. Let us see how the above listed course of actions can manifest in this scenario.

1. Minimizing error by revising the mental model (perceptual inference: you change your mind to match the world):

- Initial prediction: "My keys are next to the mail."
- Sensory input: You look down and see a silver object, but as you focus, you realize it's actually a stray coin.
- Prediction error: High (expectation: keys; reality:coin).
- The iteration: Your brain takes that error signal and sends it back up the hierarchy. It updates the mental model: "The keys are not by the mail."
- Result: You adjust your gaze and scan further to the left. Your "prediction" of where the keys are evolves based on the incoming visual data until the error is zeroed out (you finally see the keys).

2. Minimizing error by changing the physical world (active inference: you change the world to match your mind, i.e., you use action to fulfill your original prediction):

- Initial prediction: "My keys are on the table, visible and ready to be grabbed."

- Sensory input: You look at the table, but you only see a pile of newspapers. You don't see the keys.
- Prediction error: High (expectation: visible keys; reality: obscured surface).
- The Iteration: Instead of concluding "the keys aren't here," your brain initiates an action to make the prediction true. You reach out and move the newspapers.
- Result: By physically altering the environment (moving the paper), you uncover the keys. Now, your sensory input (seeing the keys) matches your original mental model. You have "acted out" the error.

It is important to note, that PEM is not only a formal apparatus which allows generating precise quantitative models, but it is also a general framework that can be applied in the service of different explanatory programs. Thus, this framework can be enriched with additional assumptions about a specific domain.

In his last research work, Uri Leron and colleagues have applied this model to the domain of data mining (Sorokina et al., 2023). The PEM process involves receiving input, focusing attention (possibly based on a goal), and then iteratively creating a model and respective predictions, testing predictions, minimizing errors and eventually, when the error is small enough, acting upon the prediction. We devised a model, based on PEM, for data mining in which we mapped between PEM steps and data mining operations, considering what operations could accomplish each PEM step and what PEM step could correspond to each data mining operation. We have proceeded to examine our proposed model empirically, and found that while not all observed data miners go through the full set of operations parallel to PEM, those who do, achieve significantly higher success in their data mining task.

3 Interdisciplinary Research Applications of Predictive Processing

3.1 The Mallet and the Model: A Wonderland of Reinforcement Learning – Benny Bornfeld

Alice lifted her flamingo mallet to strike, but the hedgehog ball uncoiled and shuffled away before the blow could land. Every time she neared a goal, the living card-soldier arches shifted to block her path. In Wonderland, the rules of the game — much like the laws of logic and physics, are not only bizarre but perpetually in flux. This collapse of the internal model is not just a literary device, it is a perfect window into how both biological and artificial minds struggle to navigate a volatile world.

Traditionally, we gather knowledge through two primary channels: the detached observation of "reading and watching" and the visceral friction of direct experience. While Large Language Models (LLMs) mirror this through a combination of massive pre-training (observation) and Reinforcement Learning from Human Feedback (experience), the true "science of optimal behaviour" lies in how an agent handles that direct interaction. This is the domain of *Reinforcement Learning (RL)* (Sutton et al, 1998).

While formal RL roots trace back to B.F. Skinner's 1930s operant conditioning and Richard Bellman's 1960s mathematics, the mechanism is as old as evolution itself. Life has always been shaped by the binary signals of "rewards" like dopamine and "penalties" like pain. As Heraclitus observed, "Everything flows, nothing stands still," and in a world that refuses to stay put, we cannot rely solely on static historical records. An agent, whether a girl in a surreal garden or a cleaning robot in a messy room, must continuously probe its environment to discover the current consequences of its actions.

To understand how this works in practice, we can map the Reinforcement Learning framework (see Fig.1) onto a simple, concrete example: a cleaning robot tasked with tidying a cluttered living room. This setup provides a clear taxonomy for the mechanics of experience:

- **The Agent:** This is the "brain" — the underlying algorithm making decisions.
- **The Environment:** The room's physical layout, the furniture, and the robot's own mechanical constraints.
- **The Rewards:** The feedback loop that shapes behaviour. The agent receives "points" for collecting litter (Positive Reward) and "penalties" for breaking a vase or bumping into a wall (Negative Reward).

The agent's singular goal is to navigate this environment to harvest the highest possible Cumulative Reward. However, as our robot begins its task, a critical fork in the road appears: Model-Based versus Model-Free learning.

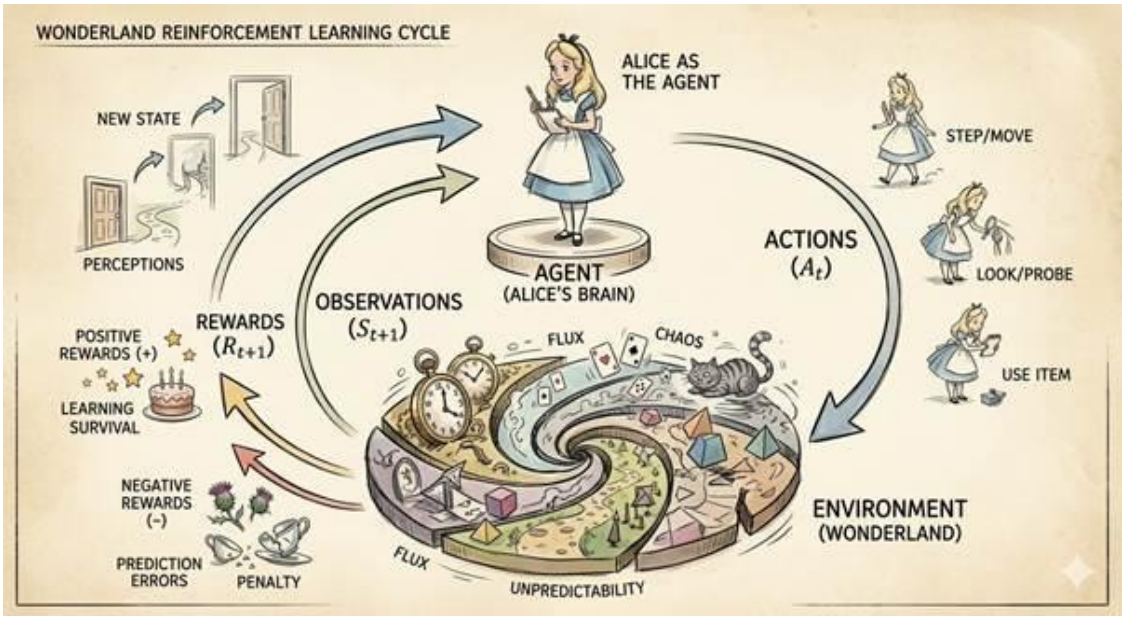


Fig 1. The RL Classic Model, Demonstrated on Wonderland (source https://gemini.google.com/share/f5b5e57373d7)

A Model-Based approach equips the robot with "priors", an internal map or a set of physical laws that allow it to simulate outcomes before moving. Much like the *Predictive Processing* theory in cognitive science, the robot can "think" ahead, pruning dangerous actions from its plan without having to hit a wall (Botvinick et al, 2014).

But this efficiency comes with a catch: the "advice" of the model is only as good as its accuracy. In the volatile shift from a predictable lab to a chaotic home, or from a Victorian garden to Wonderland, a rigid internal model can become a liability. When the world changes faster than the model can update, the agent must rely on *Model-Free* learning: abandoning the "map" to learn directly from the raw, real-time friction of its own successes and failures.

Ultimately, Predictive Processing and Model-Based RL are two sides of the same coin. Both are incredibly powerful when the internal model is precise, but they lead to a cascade of "prediction errors" when the world turns volatile. Alice's famous lament, *"give myself very good advice, but I seldom follow it"* is a profound survival strategy. When your internal model, your "advice" is based on a world that no longer exists, the only way to adapt is to ignore the model and surrender to the raw, real-time feedback of Model-Free experience.

3.2 Error Prediction as a Developmental Neurocognitive Mechanism: Executive Functions, Reading, and Plasticity in Childhood – Dalia Braverman-Jaiven and Tzipi Horowitz-Kraus

Predictive processing frameworks view cognition as a ranking system that continuously generates predictions about incoming sensory input and updates based on prediction errors — mismatches between expected and actual outcomes (Gehring, Goss, Coles, Meyer, & Donchin, 1993). Within this framework, error signals are not failures of the system but rather its primary learning currency. However, most theoretical discussions of predictive processing focus on adult cognition and abstract computational principles, with relatively little attention to how prediction error mechanisms develop, how they are instantiated in the immature brain, and how they shape learning across childhood. In this section, we argue that error prediction should be understood as a developmentally constrained, multimodal, and experience-dependent mechanism, supported neurobiologically through error monitoring systems and executive functions (EF)- are higher-order cognitive processes to manage emotions, thoughts and behavior-, and critically involved in reading development and its disorders. In short, EF are part of the brain management system.

3.2.1 Neural Error Detection as Prediction Error Signaling

At the neurobiological level, prediction errors are commonly measured through error-related event-related potentials (ERPs), most prominently the error-related negativity (ERN) and the correct-related negativity (CRN). These components reflect rapid neural responses to discrepancies between intended and executed actions and are generated primarily within medial frontal structures, including the anterior cingulate cortex (Gehring et al., 1993). The ERN has been extensively linked to performance monitoring, conflict detection, and adaptive behavioral control (Botvinick, Braver, Barch, Carter, & Cohen, 2001; Holroyd & Coles, 2002). From a predictive processing perspective, the ERN can be conceptualized as a neural signature of prediction error — signaling a mismatch between internal predictions and external feedback.

Crucially, error-monitoring mechanisms are not static across development. The magnitude, timing, and functional integration of ERN signals change throughout childhood and adolescence, matching the maturation of executive control systems (Diamond, 2013). Thus, prediction error signaling must be understood within a developmental framework in which the very mechanisms responsible for detecting and utilizing errors are themselves undergoing refinement.

3.2.2 Reading as a Predictive and Multimodal System

Reading provides a unique area for examining prediction error mechanisms in development. Fluent reading depends on the rapid and precise integration of visual, phonological, and semantic information (Breznitz, 2006), all of which rely on predictive processes operating across multiple temporal scales (Breznitz & Misra, 2003). Skilled readers continuously generate expectations about upcoming letters, sounds, and words, updating these predictions as new information becomes available (Share, 1995). Prediction errors arise when expected phonological or semantic representations fail to match the visual input, leading to corrective updating (Horowitz-Kraus & Breznitz, 2008).

While traditional models of reading have emphasized decoding and language comprehension, they often overlook the contribution of executive functions and error monitoring (Gough, 1986). However, growing evidence indicates that reading fluency relies on EF processes such as attentional control, working memory, inhibition, and self-monitoring (Cirino et al., 2019; Horowitz-Kraus, 2023; Kieffer & Christodoulou, 2020). Within a predictive processing framework, these executive systems play a critical role in regulating the precision of prediction errors — determining which mismatches should incite learning and which should be ignored (Horowitz-Kraus & Breznitz, 2008).

3.2.3 Dyslexia as Disrupted Predictive Tuning

Developmental dyslexia clearly illustrates how impaired processing of prediction errors can disrupt learning. Children with dyslexia typically show slow and inaccurate reading, alongside deficits in phonological processing, attention, and processing speed. Neurobiological evidence further indicates impairments in error-monitoring mechanisms. Specifically, reduced ERN amplitudes have been observed in children with dyslexia during reading-related tasks, suggesting diminished sensitivity to prediction mismatches (Breznitz et al., 2013; Horowitz-Kraus, 2012; Horowitz-Kraus & Breznitz, 2013).

Rather than reflecting on a single deficit, these findings support an asynchrony-based account of dyslexia, in which prediction and feedback signals across sensory and executive systems are poorly coordinated. From this perspective, reading difficulties do not stem from an excess of errors, but from a reduced capacity to use prediction errors for adaptive learning. Prediction signals may be temporally misaligned or insufficiently weighted, limiting their contribution to learning.

3.2.4 Executive Functions, Audiovisual Integration, and Error Utilization

Executive functions are closely linked to audiovisual integration (AVI), a core process supporting fluent reading (Zhou, Cheung & Chan, 2020). EF systems regulate attention,

coordinate timing between auditory and visual inputs, and support the detection and correction of errors. Disruptions in AVI have been repeatedly documented in children with reading difficulties and are associated with reduced reading fluency (Karipidis et al., 2017).

Neuroimaging studies using EEG and functional MRI show dynamic interactions between EF networks — particularly the cingulo-opercular and frontoparietal networks — and auditory and visual regions during reading tasks and at rest (Farah et al., 2024; Horowitz-Kraus, & Holland, 2015). These interactions are especially relevant for prediction error processing, as they enable the rapid detection of mismatches and the allocation of cognitive resources necessary for correcting them.

Plasticity Through Predictive Recalibration

Intervention studies provide critical evidence that prediction error mechanisms are plastic and modifiable. Visual Rhythmic Reading Training (RRT), which manipulates the presentation of text by gradually deleting letters at an individually calibrated pace, has been shown to improve reading fluency and comprehension in children with dyslexia (Horowitz-Kraus, 2014). From a predictive processing perspective, this intervention can be understood as recalibrating the temporal alignment between predictions and sensory input, thereby narrowing the window for error detection and updating.

Neurobiological evidence indicates that such training leads to increased ERN amplitudes, enhanced functional connectivity within executive networks, and stronger connection between auditory and visual systems — effects observed not only during reading tasks but also during resting-state measurements (Horowitz-Kraus, 2016; Horowitz-Kraus & Holland, 2015; Horowitz-Kraus et al., 2025). Importantly, short-term gains in EF have been shown to moderate long-term improvements in reading fluency, highlighting the central role of EF-mediated error utilization in sustained learning (Taran et al., 2025).

3.2.5 Toward a Multidimensional Account of Error Prediction

Taken together, these findings support a multidimensional description of error prediction that integrates neurobiological, developmental, and educational perspectives. Prediction errors are not abstract computational signals but are embedded within sensory, cognitive, and social contexts that shape their effectiveness. During development, successful learning depends not on minimizing errors, but on finding a balance in which errors are timely, informative, and appropriately processed.

Theoretically, this perspective highlights the need for predictive models that account for the developmental constraints and specialized learning systems. Practically, it suggests that educational interventions should focus on shaping environments that support effective error utilization rather than simply reducing task difficulty or error frequency. Error prediction,

when properly tuned, serves as a powerful engine for learning, adaptation, and cognitive growth.

4 Explaining Emotional Experiences via Predictive Processing

4.1 To Predict or Not to Predict? Prediction Error as Aesthetic Pleasure – Yuvalal Liron

A close-up fills the screen: a woman's eyes, welling with tears. What do you assume? A broken heart, perhaps. A loss. Already, without effort, you have begun constructing a model: who is this person, what has happened to her, what kind of film is this? The camera pulls back slowly, and we see that she is standing in a kitchen, chopping vegetables. Ah — she is crying because she is cutting onions. We smile at ourselves: we were fooled. The camera moves in closer. She is actually chopping cucumbers. The tears are real after all.

I saw this opening scene in a film nearly thirty years ago. I cannot remember which film, and I have not been able to find it. I may be misremembering, or even inventing. But I have carried the experience of those reversals for three decades, which is itself a point worth noting: what I remember is not the film, but the feeling of my model being overturned, three times in under a minute. Prediction error, it seems, is what makes things stick.

This is predictive processing at work, but not in the way the framework typically describes it. The standard account emphasizes that the brain generates predictions about incoming information, and that learning occurs when prediction errors drive the system to update its models. The emphasis falls on *minimization*: the brain wants to reduce surprise, to make the world more predictable. And yet, as viewers, readers, and listeners, we actively seek out experiences designed to surprise us. We pay money for it. We hate spoilers precisely because they eliminate the prediction errors we were looking forward to. The standard account, then, tells only part of the story. In aesthetic experience, prediction error is not a cost to be minimized. It is a source of pleasure. Good art lives in the balance between the satisfaction of getting it right ("I knew it!") and the delight of getting it wrong.

Consider Ibsen's *A Doll's House* (1879). The play opens with Nora Helmer arriving home. She has no key of her own; someone must let her in. She is in a bright, playful mood. She sneaks through the room, and we discover she is hiding a bag of macaroons. Notice how little information we need. Five details, and we have already begun assembling an entire world: a marriage with unequal power, a woman who hides small pleasures, a household with rules she navigates around. We cannot not predict. Our system starts building a model from almost nothing, filling in character, relationships, and stakes before the first real scene has begun.

Ibsen lets us keep building that model for three full acts. And then he demolishes it. In the final scene, Nora tells her husband she is leaving. Leaving him, leaving the children, leaving the entire structure of the life we have watched her inhabit. For the original audience, the shock ran deeper

than plot. The theatrical convention of the era, the *well-made play*, promised that whatever disruptions arose, order would be restored by the final curtain. Ibsen broke that promise. The prediction error operated not at the level of events, but at the level of dramatic convention itself. And its force was proportional to the investment: three acts of confident model-building made the collapse all the more powerful.

But here is the deeper puzzle. What happens when the audience already knows the ending?

The ancient Greeks went to the theater knowing their myths. Every spectator watching *Oedipus Rex* knew that Oedipus himself was the killer he was hunting. There was no plot-level surprise to speak of. And yet Sophocles' drama generates extraordinary tension. How?

The answer, I believe, lies in the fact that prediction operates at multiple levels simultaneously. Even when the *what* is fully predicted, the *how* and *when* remain open. The audience knows the destination but not the route. Each scene poses its own questions: how will Oedipus react to this piece of evidence? When will the recognition come? How will Jocasta respond? The dramatic tension is not about the outcome but about the unfolding.

This multi-level structure turns out to be remarkably durable. It is, in essence, the same mechanism that drives the modern romantic comedy. Every viewer of a romcom knows, with near-certainty, that the couple will end up together. No one watches for the *what*. We watch for the *how*: the misunderstandings, the obstacles, the moment of apparent failure before the reunion. From Sophocles to romantic comedy, the architecture is the same — a predicted frame filled with unpredicted details.

One final example, this time a comedy. In Billy Wilder's *Some Like It Hot* (1959), two jazz musicians flee the mob by disguising themselves as women in an all-female band. Jerry, played by Jack Lemmon, adopts the persona of "Daphne" and inadvertently attracts the affections of Osgood, an elderly millionaire who proposes marriage. In the film's famous final scene, Jerry tries to extricate himself. He offers objections, one after another, each of which should, by any reasonable prediction, end the conversation. Osgood absorbs each new piece of information and refuses to update his model. When Jerry finally pulls off his wig and announces "I'm a man!", Osgood's response — "Well, nobody's perfect" — is one of the great comic lines in cinema, precisely because it overturns every prediction the audience has built about how this revelation must go. We laugh because our predictive model, and Jerry's, collapses simultaneously.

These examples suggest that when we turn the lens of predictive processing toward aesthetic experience, something new comes into view. Art does not merely tolerate prediction error; it *designs* for it. The artist's craft consists, in large part, of managing the audience's predictions: building models in order to break them, confirming expectations in order to overturn them, and calibrating the balance between the two.

This resonates with a theme that ran through Uri Leron's work in mathematics and computer science education. Uri argued that learners' errors and intuitions are not obstacles to understanding but essential starting points, the raw material from which meaning is constructed. The same principle

applies to aesthetic experience. In art as in learning, the goal is not to eliminate prediction error but to make it *productive*: to create conditions in which the gap between expectation and reality becomes a source not of threat, but of pleasure, insight, and understanding.

4.2 A Window into My Grandfather's Mind: Predictive Play, Humor, and the Beauty of Models – Yonathan Bornfeld

When I think about my grandfather, the first thing that comes to mind isn't just the stories he told or the lessons he imparted — it's the way his mind worked. Everything about him, from his humor to the way he approached mathematics and people, felt like a lens into a deeply playful, intricate, and thoughtful process. I want to share these memories not as objective facts, but through my own eyes, as a reflection of how his mind left an impression on me — and, in many ways, how it mirrors the ideas behind predictive processing, a theory that suggests our brains are constantly generating and updating models of the world.

Humor was, for me, the most immediate and vivid expression of my grandfather's mind. If I were to start anywhere, it would be here. His humor was a delicate balance between predictability and surprise. On the one hand, you could almost always anticipate that he would make a funny comment. Over time, we developed a shared "model" of the categories his humor might fall into, and many of us could guess which one was coming next. Yet, even within these bounds, there was always something irreducibly unique in how he delivered it — the tone of his voice, the precise expression on his face, the timing of the pause — that no model could fully capture. We would often find ourselves recounting a funny thing he had said, thinking we could predict it, but only partially succeeding. In his own words, he described his sense of humor as "anarchism — it's about breaking frameworks." And indeed, in his humor, he broke patterns, defied expectations, and created a joyful uncertainty that left us laughing and thinking simultaneously.

This interplay between expectation and novelty is strikingly reminiscent of predictive processing. In this framework, our brains are constantly generating predictions about what we expect to happen and adjusting them when reality surprises us. My grandfather's humor seemed to operate on both levels: it played with our internal models — our expectations of what he might say — while simultaneously introducing elements that could not be fully predicted. It was an elegant, human illustration of the balance between prediction and surprise.

But his fascination with models extended far beyond humor. I realized that a common thread in his interests — from mathematics to human interactions — was his delight in consciously building models of the world. Unlike the implicit, survival-oriented modeling that evolution equipped our brains for — a primordial human predicting the dangers of the forest — my grandfather's models were deliberate, reflective, and, in many cases, playful.

He would not just model how to survive; he modeled social dynamics, abstract problems, and even the behavior of his cats, imagining himself as an observer "above" their little feline society. He even approached mundane tasks like arranging the dishwasher as an opportunity for refinement and aesthetic improvement, elevating everyday routines to exercises in design and understanding.

Mathematics, perhaps unsurprisingly, was one of the clearest examples of this modeling mindset. From early on, mathematics framed our relationship. We would rise early, wander through quiet streets, and discuss mathematical ideas, side by side. There were two forms of learning I experienced: the structured, efficient method of school lessons, and the more exploratory, layered approach my grandfather embodied. The school model provided the precise tools needed to solve problems — a functional and reliable approach. My grandfather's model started elsewhere, growing gradually, embedding multiple layers of understanding. The structured model gave immediate strategies, while his deeper, exploratory approach cultivated a richer, more enduring comprehension. It was less about immediate efficiency and more about opening a window into the beauty and complexity of the mathematical landscape. Even in his mathematics club, held at our family home, he delighted in sharing these "gems" of thought with visitors, nurturing a love of depth, nuance, and elegance.

The modeling extended seamlessly into his interactions with people. He approached social situations as carefully as he did mathematical problems, seeking fairness, transparency, and elegance in human affairs. For instance, when selecting graduate students for his supervision, he would meet with candidates, discuss their interests, and then assign reading tasks with the expectation that only those genuinely engaged would return with reflections. Around eighty percent did not follow through, effectively filtering themselves while ensuring the process was equitable. His models of human behavior were dynamic and multilayered: he tried to imagine how others perceived the world, including their internal models, and he sought solutions that were simultaneously effective, fair, and elegant. This attentiveness to others' perspectives resonates with predictive processing as well, which emphasizes the importance of continuously updating internal models based on feedback from the environment — in this case, social signals.

Looking back, the most enduring imprint my grandfather left was his commitment to *depth and understanding*, his quest for elegance, and his love of the world's complexity. Whether modeling mathematical problems, human behavior, or the playful social lives of cats, his approach was never purely utilitarian or survival-oriented. It was about appreciating the underlying structures, the patterns, and the beauty inherent in the systems he explored. In predictive processing terms, he exemplified an extraordinary meta-awareness of modeling itself — constantly refining, reflecting, and delighting in both the predictive and the surprising aspects of life.

In many ways, reflecting on my grandfather's life feels like exploring the workings of a mind that was constantly experimenting with prediction and surprise. His humor taught me the joy of balancing expectations and novelty; his mathematics taught me to value layered understanding over immediate correctness; and his human insight taught me to consider fairness, perspective, and elegance in social interactions. All of these domains, at heart, involved constructing internal models, testing them against reality, and adjusting them — a deeply human embodiment of predictive processing.

Ultimately, what I carry with me is the sense of wonder that comes from engaging with someone who loved to think about thinking, who enjoyed building models not out of necessity, but out of curiosity, creativity, and delight. My grandfather's life offers a window into the art of prediction — not just as a cognitive tool, but as a form of living fully, attentively, and joyfully in the world.

5 Conclusion

Our exploration of Predictive Processing serves as a profound testament to the enduring values that defined the life and work of Uri Leron. By weaving together insights from data science, machine learning, neuroscience, aesthetics, and humor, this paper demonstrates that interdisciplinarity is not merely a method of research, but a way of seeing the world in its full, interconnected complexity.

At the heart of this framework is Leron's commitment to bridge building - framing learning as a gradual mediation between intuitive and analytical modes of understanding. Through the lens of Prediction Error Minimization (PEM), we see that the surprise stemming from a mismatched expectation is not a deficit to be corrected, but a productive error: an essential anchor for meaning-making and the primary engine for cognitive growth.

Leron's quest was always for elegance, whether in a mathematical proof, a social interaction, or the playful social dynamics of a cat. He taught us that expertise is not defined by the absence of errors, but by the sophisticated capacity to transform those errors into "gems" of insight and opportunities for deeper understanding. His legacy reminds us that building models of the world is a creative, joyful act - a form of living fully and attentively in the beautiful space between what we expect and what we discover.

We, his followers, will keep his spirit and inspiration alive, by breaking frameworks, building bridges, and finding the beauty in every error.

6 References

Botvinick, M. M., Braver, T. S., Barch, D. M., Carter, C. S., & Cohen, J. D. (2001). Conflict monitoring and cognitive control. *Psychological review*, *108*(3), 624.

Breznitz, Z. (2006). *Fluency in reading: Synchronization of processes*: Routledge.

Breznitz, Z., & Misra, M. (2003). Speed of processing of the visual-orthographic and auditory-phonological systems in adult dyslexics: the contribution of "asynchrony" to word recognition deficits. *Brain Lang*, *85*(3), 486-502. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/12744959

Breznitz, Z., Shaul, S., Horowitz-Kraus, T., Sela, I., Nevat, M., & Karni, A. (2013). Enhanced reading by training with imposed time constraint in typical and dyslexic adults. *Nature communications*, *4*(1), 1-6.

Cirino, P. T., Miciak, J., Ahmed, Y., Barnes, M. A., Taylor, W. P., & Gerst, E. H. (2019). Executive function: association with multiple reading skills. *Reading and Writing*, *32*(7), 1819-1846. doi:10.1007/s11145-018-9923-9

Clark, A. (2016). Busting Out: Predictive Brains, Embodied Minds, and the Puzzle of the Evidentiary Veil. Noûs 51, 727–753. https://doi.org/10.1111/nous.12140 13.

Clark, A. (2018). A nice surprise? Predictive processing and the active pursuit of novelty. Phenom Cogn Sci 17, 521–534 (2018). https://doi.org/10.1007/s11097-017-9525-z

de Bruin, L., Michael, J. (2017). Prediction error minimization: implications for embodied cognition and the extended mind hypothesis. Brain Cogn. 112, 58–63. https://doi.org/10.1016/ j.bandc.2016.01.009 14.

de Bruin, L., Michael, J. (2018). Prediction error minimization as a framework for social cognition research. Erkenntnis 86(1), 1–20. https://doi.org/10.1007/s10670-018-0090-9 16.

Diamond, A. (2013). Executive functions. *Annual review of psychology*, *64*(1), 135-168.

Dubinsky, E., Dautermann, J, Leron, U. and Zazkis, R.. (1995). On learning fundamental concepts of group theory, Educational studies in Mathematics 27 (3), 267-305.

Farah, R., Dworetsky, A., Coalson, T., Petersen, S. E., Schlaggar, B. L., & Horowitz-Kraus, T. (2024). Enhanced auditory–visual–executive network coupling following reading intervention in children. *Cerebral Cortex, Advance online publication*.

Gehring, W. J., Goss, B., Coles, M. G., Meyer, D. E., & Donchin, E. (1993). A neural system for error detection and compensation. *Psychological Science*, *4*(6), 385-390.

Gough, P. B., & Tunmer, W. E. (1986). Decoding, reading, and reading disability. *Remedial and Special Education*, *7*, 6-10.

Hadar, I. and Leron, U. (2008) "How Intuitive is Object Oriented Design?", Communications of the ACM, vol. 51(5), pp. 41-46.

Hazzan, O., Zazkis, R., Levy, M., Hadar, I. and Buchbinder, O. (2024). An Ode to a Mathematician: Commemorating Uri Leron, Digital Experiences in Mathematics Education. https://doi.org/10.1007/s40751-024-00145-w

Holroyd, C. B., & Coles, M. G. (2002). The neural basis of human error processing: reinforcement learning, dopamine, and the error-related negativity. *Psychological review*, *109*(4), 679.

Horowitz-Kraus, T. (2012). The Error Detection Mechanism Among Dyslexic and Skilled Readers: Characterization and Plasticity. In Z. Breznitz, O. Rubinsten, V. J. Molfese,

& D. L. Molfese (Eds.), *Reading, Writing, Mathematics and the Developing Brain: Listening to Many Voices* (Vol. 6, pp. 113-130): Dordrecht: Springer.

Horowitz-Kraus, T. (2014). Reading fluency training improves neural activity in children with dyslexia. *Brain and Behavior, 4*(6), 828-843.

Horowitz-Kraus, T. (2016). Increased error-related negativity following reading intervention in children with dyslexia. *Dyslexia, 22*(1), 67-80.

Horowitz-Kraus, T. (2023). The role of executive functions in fluent reading: Lessons from reading acquisition and remediation. *MIND, BRAIN, AND EDUCATION, 17*(4), 373-382.

Horowitz-Kraus, T., et al. (2025). Training-induced plasticity of audiovisual and executive networks in children with reading difficulties. *Network Neuroscience, Advance online publication*.

Horowitz-Kraus, T., & Breznitz, Z. (2008). An error-detection mechanism in reading among dyslexic and regular readers–An ERP study. *Clinical Neurophysiology, 119*(10), 2238-2246.

Horowitz-Kraus, T., & Breznitz, Z. (2013). Compensated dyslexics have a more efficient error detection system than noncompensated dyslexics. *Journal of Child Neurology, 28*(10), 1266-1276.

Horowitz-Kraus, T., & Holland, S. K. (2015). Greater functional connectivity between reading and executive control networks following intervention in children with dyslexia. *Annals of dyslexia, 65*(1), 1-23.

Karipidis, I., Pleisch, G., Röthlisberger, M., Hofstetter, C., Dornbierer, D., Stämpfli, P., & Brem, S. (2017). Neural initialization of audiovisual integration in prereaders at varying risk for developmental dyslexia. *Human Brain Mapping, 38*(2), 1038-1055.

Kieffer, M. J., & Christodoulou, J. A. (2020). Automaticity and control: How do executive functions and reading fluency interact in predicting reading comprehension? *Reading Research Quarterly, 55*(1), 147-166.

Leron, U. (1983). Structuring mathematical proofs, The American Mathematical Monthly 90 (3), 174-185.

Leron U. and Dubinsky, E. (1994)., An abstract algebra story, The American Mathematical Monthly 102 (3), 227-242.

Leron, U., & Ejersbo, L. R. (2021). Good errors in mathematics education. International Journal of Mathematical Education in Science and Technology, 52(5), 752–766. https://doi.org/10.1080/0020739X.2019.1706004

Leron, U. & Hazzan, O. (2006). The Rationality Debate: Application of Cognitive Psychology to Mathematics Education. Educ Stud Math 62, 105–126. https://doi.org/10.1007/s10649-006-4833-1.

Leron, U. & Hazzan, O. (2009). Intuitive vs analytical thinking: four perspectives. Educ Stud Math 71, 263–278. https://doi.org/10.1007/s10649-008-9175-8.

Miller, M., Clark, A. & Schlicht, T. (2022). Editorial: Predictive Processing and Consciousness. Rev.Phil.Psych. 13, 797–808. https://doi.org/10.1007/s13164-022-00666-6

Share, D. L. (1995). Phonological recoding and self-teaching: sine qua non of reading acquisition. *Cognition, 55*(2), 151-218; discussion 219-126. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/7789090

Sorokina, E., Soffer, P., Hadar, I., Leron, U., Zerbato, F., Weber, B. (2023). "PEM4PPM: A Cognitive Perspective on the Process of Process Mining," International Conference on Business Process Management 2023.

Taran, E., Farah, R., Rosch, K. S., & Horowitz-Kraus, T. (2025). Executive functions moderate long-term reading fluency outcomes following intervention. *Dyslexia, Advance Online Publication*.

Zhou, H.-y., Cheung, E. F., & Chan, R. C. (2020). Audiovisual temporal integration: Cognitive processing, neural mechanisms, developmental trajectory and potential interventions. *Neuropsychologia, 140*, 107396.

Botvinick, Matthew, and Ari Weinstein. "Model-based hierarchical reinforcement learning and human action control." Philosophical Transactions of the Royal Society B: *Biological Sciences 369.1655 (2014).*

Sutton, Richard S., and Andrew G. Barto. Introduction to reinforcement learning. *Vol. 135. Cambridge: MIT press, 1998.*